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3-People non-zero-sum games and geometric presentation on game theory

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Abstract

The game theory that provides us an optimum decision option in such a position an interaction decision is given takes place more often both in our daily life and business life. The interest in this issue is increasing when the consistency between the results of the application and application territory is seen. It will be seen that we come close to a period that theory will be used more often.

In this research it has been given fundamental concept of game theory and it has been given examples on-non sum-zero game that is for three people.

Keywords: Game theory, strategy

1. Introduction

The biggest problem that the business management encounters is the process of deciding in economic field. So, the decisions made by the businesses as producers or consumers directly affect the production or consumption types. Businesses generally aim to reach the prudential targets they have determined depending on their internal conditions in the process of deciding. The businesses, thus, choose to predict the future with the data obtained through previous periods and with the quantitative deciding techniques such as mathematical programming (Operations research), cross section data regression models, time series trend analysis in the process of deciding.

In these methods, the mutual interactions between the variables are not mostly considered or it is accepted that this is reflected in the models formed automatically. Besides, several difficulties are experienced in the addition of many socio-economic variables to the model in the results obtained from these mentioned quantitative variables (OZDIL 1998).

Game theory is a powerful managerial tool as it provides a beginning for the solution in the process of complex interactive decision making. Game theory gives the answer to the question "What should be the optimal strategy in the purpose of minimization if the loss in question, or maximization if the gain in question?" for the competitive decision-maker no matter what strategy the opponent play. In this field, the game theory can provide a good score in making economic decisions in the economic markets where competition takes place. The game theory is a mathematical approach that analyzes the deciding process considering the deciding process of the opponents in clash environments. The question "Without knowing which behavior the opponents will choose, what should be the most rational behavior to make positive move decisions" caused this theorem to be raised. Thus, the Game Theory is a mathematical approach that explains the struggle of the complex wheels (OZDIL 1998).

To meet the analysis needs of conflict situations, special mathematical techniques named as theory of games have been developed. The purpose of this theory is to analyze the most rational movement ways of the both parties which are against each other. As there are several factors, real life conflict situations are extremely complex and quiet hard to be analyzed. Hence, to make a mathematical analyzes possible, we need to remove base factors and create simplified models. These models are called games (VENTSELL 1965).

The purpose of this study is to bring a solution method to the 3-player non-zero-sum games.

2. Material and method

Geometric method which was developed by Ventsell for two-player *2x2* games was used in the development of geometric method for 3-player non-zero-sum games (VENSTELL 1965).

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2.1. The geometric method for 2x2 games

For the solution of a *2x2* game, a simple geometric interpretation can be given. The *2x2* game whose matrix can be seen on the side is considered and the diagram below is drawn on *xy* plane. Our strategy is shown on *x* axis. A_1 strategy is indicated with x=0 and A_2 is indicated with *x=1*. I and II perpendiculars are drawn from A_1 and A_2 points. For A_1 strategy, the gains are marked on I axis, and for A_2 strategy they are marked on II axis.

Table 2.1. B_1 and B_2 strategies.

A/B	B_1	B_2
a_1	a_{11}	a_{12}
a_2	a_{21}	a_{22}

First of all, it is accepted that our opponent uses B_1 strategy. This defines a point whose ordinate is a_{11} on I-I and a point whose ordinate is a_{21} on II.

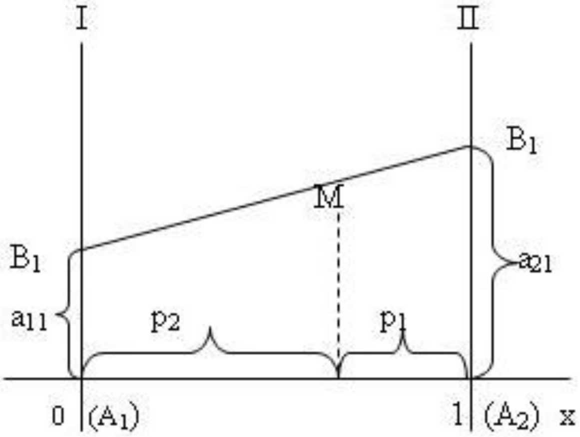

Figure 2.1. The geometric drawing for 2x2 game (VENSTELL 1965).

These points define B_1B_1 line. If we use

$$S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & p_2 \end{pmatrix}$$

the mixed strategy against our opponent's B_1 strategy

$$a_1\, p_1 + a_2\, p_2$$

The average gain is given with the ordinate of *M* point whose abscissa is p_2 on B_1 B_1 line. We will call B_1 B_1 line as B_1 which shows the gains for B_1 strategy. B_2 strategy is drawn completely the same.

We want to find a S_A^* optimal strategy, that is, in this strategy, minimum gain will be maximum gain for any B_1 strategy. To do this, lower bound is drawn for B_1 and

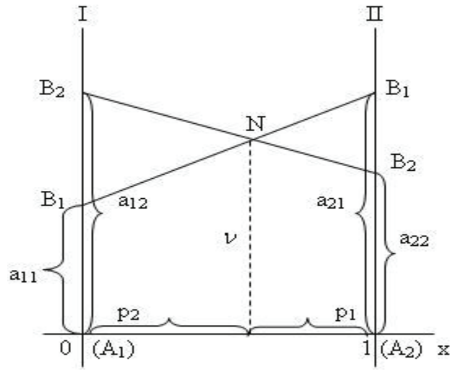

Figure 2.2. The geometric drawing for 2x2 game (VENSTELL, 1965).

B_2 strategies. This lower bound provides minimum gains for our all mixed strategies. The *N* point in which this minimum becomes maximum is the solution of the game.

The reasons of these drawing can be seen clearly from the general figure below. Here;

$$\frac{m - a_{11}}{p_2} = \frac{a_{21} - a_{11}}{p_1 + p_2}$$

or

$$m = \frac{a_{11}p_1 + a_{21}p_2}{p_1 + p_2}$$

When p_1+p_2=1, m=$a_{11}p_1$+$a_{21}p_2$=v

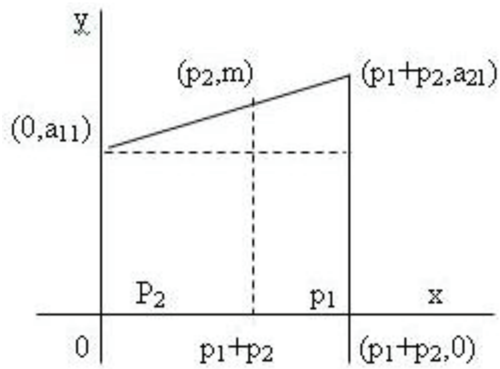

Figure 2.3. The geometric drawing for *2x2* game (VENSTELL 1965).

defines both the solution and the values. The ordinate of the *N* point is the *v* value of the game. P_2 abscissa is the fraction of A_2 strategy in our S_A^* optimal mixed strategy.

For the situation which can be seen below, it is defined with the intersection point of the solution strategies. However, the solution is not always in this point.

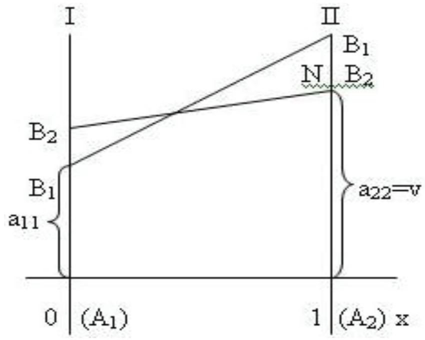

Figure 2.4. The geometric drawing for 2x2 game (VENSTELL 1965).

In the situation seen in Figure 2.4, the strategies intersect and the solution of the game is a simple strategy for each player (A_2 and B_2) and the value of the game is $v=a_{22}$. Thus, the game has a saddle point and A_2 strategy dominates A_1 strategy. No matter which strategy the opponent uses, using A_1 strategy would provide a smaller amount of gain than the A_2 strategy would provide.

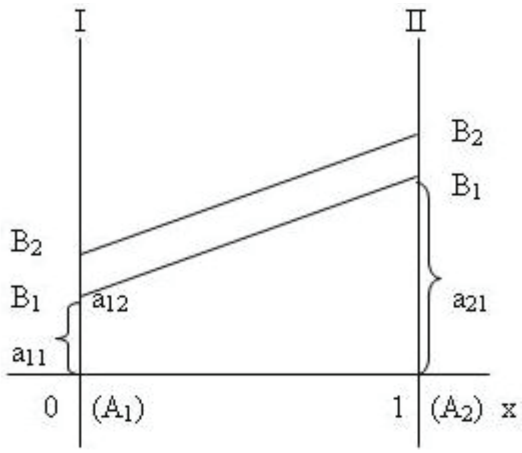

Figure 2.5. The geometric drawing for 2x2 game (VENSTELL 1965).

For the situation in which the opponent has a dominant strategy, the diagram has the figure as seen above. In this situation, lower bound is B_1 strategy which is dominant on B_2.

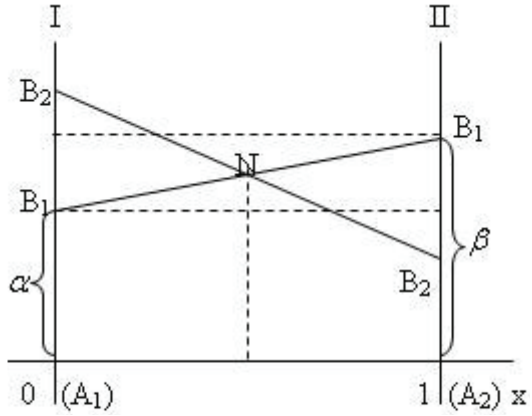

Figure 2.6. The geometric drawing for 2x2 game (VENSTELL 1965).

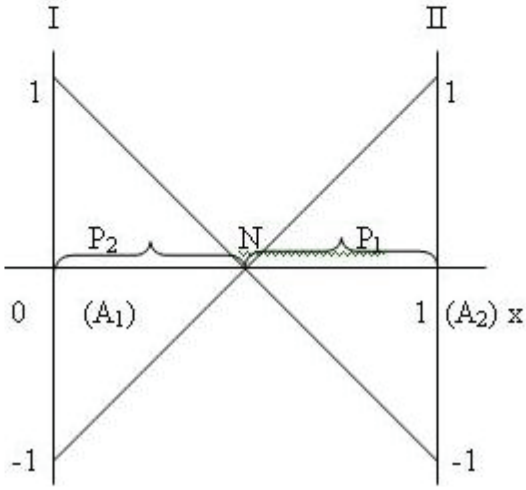

Figure 2.7. The geometric drawing for 2x2 game (VENSTELL 1965).

α lower values and *β* higher value of a game can be found from a geometric diagram.

This geometric method is further explained by giving diagrams for *2x2* games.

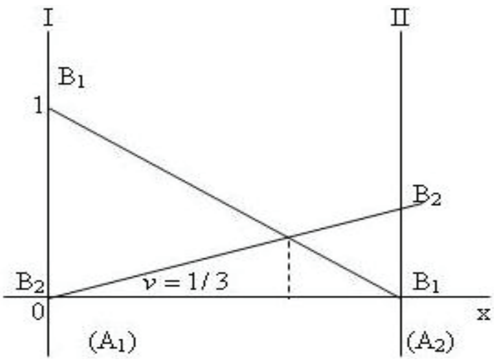

Figure 2.8. The geometric drawing for 2x2 game (VENSTELL 1965).

3. Results

3.1. 3-player non-zero-sum games and geometric presentation

We saw that any two-player game can be solved by using a simple coordinate system. We can also solve non-zero-sum games in which the number of the players becomes 3 by using the same general method by drawing it on the Cartesian coordinate system.

Let's assume that there are three sides as *A, B, C* in a non-zero-sum game. Let's consider a game in which

A's A_1, A_2 …………… A_m as *m*

B's B_1, B_2 …………… B_n as *n*

C's C_1, C_2 …………… C_k as *k*

strategies and for each strategy trinity the drawing below is drawn.

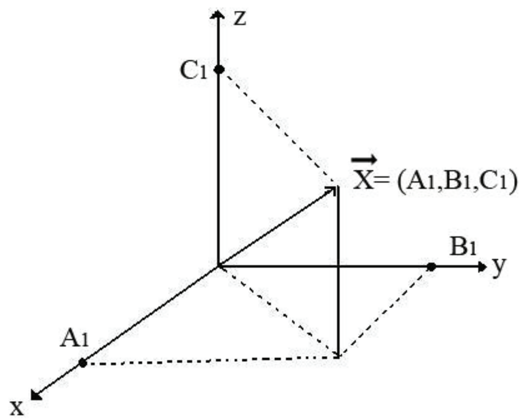

Figure 3.1. The coordinate plane for 3-player games

First of all, let's assume that A uses A_1 strategy, B uses B_1 strategy and C uses C_1 strategy.

This; for

$A = (A_1,0,0)$ $B = (0,B,0)$ $C = (0,0,C)$

can be written as. $\vec{X} = (A_1, B_1, C_1)$.

In the zero-sum games, even though the gain matrixes are always written by A, separate gains are written for each side in non-zero-sum games. The solution of the game is the strategy in which the least loss is possible for the three parts.

Now, let's analyze a 3-player conflict situation modelled example and solve it.

Example 1. When the World War I started, England was on the side of allied powers, Italy was on the side of central powers, and Greece stayed neutral. England wanted both state on its side and England promised those states İzmir and its surrounding provided that these states continued the war on England's side.

Solution:

A: England B: Greece C: Italy

A has A_1: giving İzmir and its surrounding to Italy

A_2: giving İzmir and its surrounding to Greece

B has B_1: go to war on the side of allied powers

B_2: not go to war

C has C_1: staying on the side of central powers

C_2: change its side to allied powers

two strategies as above.

The game is a 3-player *2x2x2* game. That means there are 8 possible situations.

If England gave İzmir to Italy, it would create a more powerful Italy and this could be a trouble in the future, so giving it to Greece the weaker part would be a more lucrative business. Thus;

Let's assume that while A_1 strategy brings +1 gain

A_2 strategy brings +2 gain.

B; that is, if Greece went to war in return of İzmir, it would retrain Aegean that it had longed for years; it didn't go to war and stay neutral, neither it would lost anything nor it would gain anything.

B_1: strategy brings+2

B_2: strategy brings 0.

C; for Italy, staying on the side of central powers, that is, changing its side from a powerful state like Germany to a powerful state like England would bring neither profit nor loss, but having İzmir and Aegean Island would be profitable.

C_1 brings 0

C_2 strategy brings +1.

Now, for each situation let's write down values related to strategies of all sides and draw the diagram on the coordinate plane.

***$A_1B_1C_1$*:** England gives İzmir to Italy, Italians continues to the war on the side of central powers. Greece goes to war on the side of allied powers.

A: 1
B: -1
C: -1

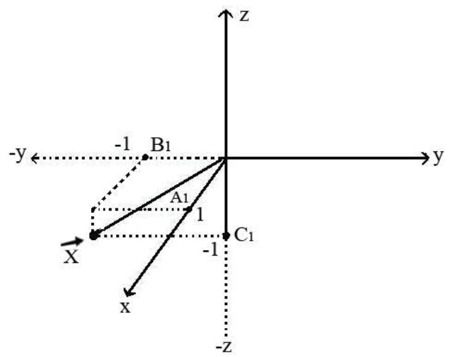

Figure 3.2. $A_1B_1C_1$ strategy.

$A_1B_1C_2$: İzmir is given to Italy, Greece joins to allied powers. Italy changes its side to allied powers.

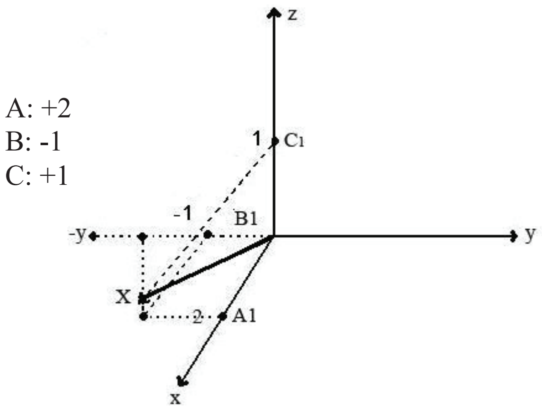

Figure 3.3. $A_1B_1C_2$ strategy.

$A_1B_2C_1$: İzmir is given to Italy, Greece does not go to war, Italy stays on the side of central powers.

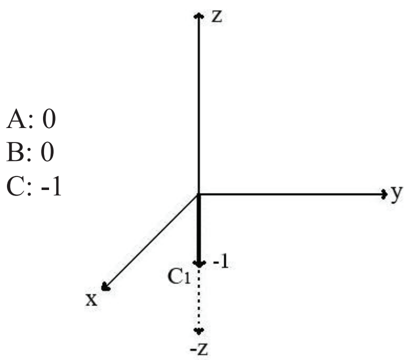

Figure 3.4. $A_1B_2C_1$ strategy.

$A_1B_2C_2$: İzmir is given to Italy, Greece does not go to war, Italy changes its sides to allied powers.

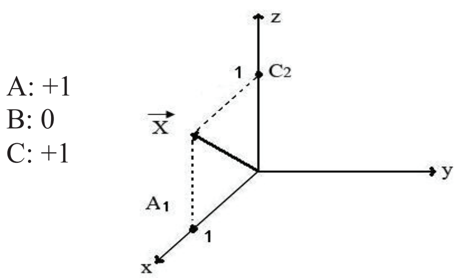

Figure 3.5. $A_2B_1C_1$ strategy.

***$A_2B_1C_1$*:** - İzmir, is given to Greece, Greece joins allied powers, Italy joins central powers.

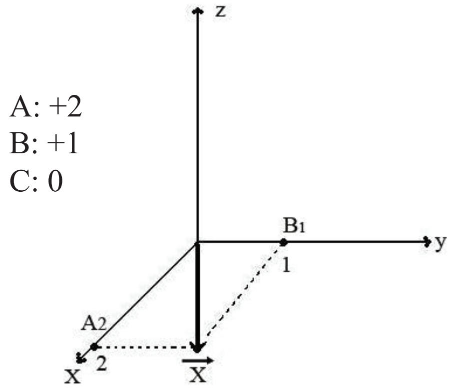

Figure 3.6. $A_2B_1C_1$ strategy.

***$A_2B_1C_2$*:** İzmir is given to Greece, Greece joins allied powers, Italy changes its side to allied powers.

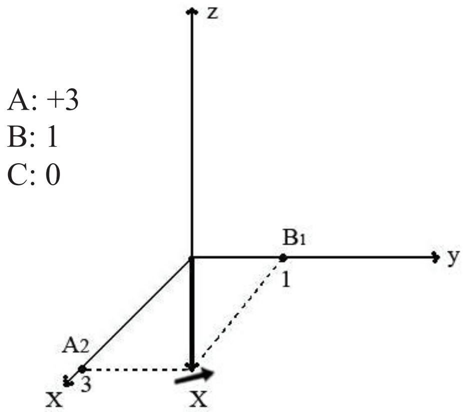

Figure 3.7. $A_2B_1C_1$ strategy.

***$A_2B_2C_1$*:** İzmir is given to Greece, Greece does not go to war, Italy stays on the side of central powers.

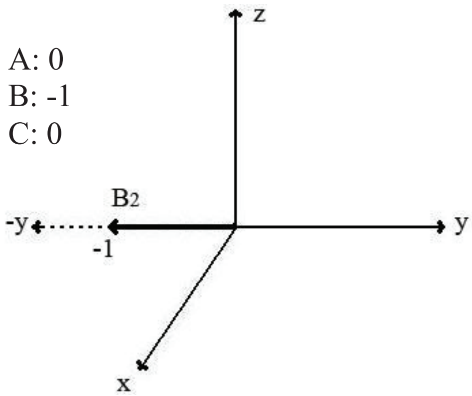

Figure 3.8. $A_2B_2C_1$ strategy.

$A_2B_2C_2$: (İzmir is given to Greece, Greece does not go to war, Italy stays on the side of central powers.)

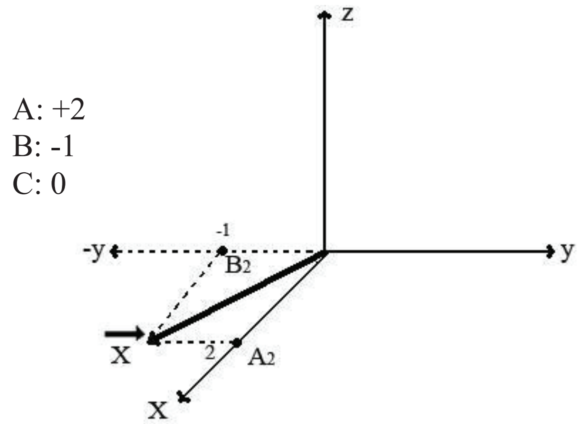

Figure 3.9. $A_2B_2C_2$ strategy.

For the solution of the game, each party should choose strategies that would provide them the least loss. In the World War I, $A_2B_1C_1$ strategy was applied and thus England was the state which benefited more from this.

In this 3-player game, the situation in which the most suitable strategies that Nash equilibrium has not unbalanced for the three party were chosen (NASH 1950).

It is not possible to talk about 3-player games for zero-sum games.

The loss of the winning party in zero-sum games is equal to the gain of the other party. In 3-player games, there are not only us and our opponent, but a third party is also in the game and the game has now become I– you– he. Even if we distribute the profit equally, my loss will be higher than the other parties' gain.

A side *x* profit

B side *x* profit

C side *x* profit

The loss of *B* is *2x*, the profits of the opponents are *x* lira. As 2x ≠ x, the game is not zero-sum.

4. Discussion and conclusion

NASH (1950) introduced his nobel-winning work - the balance of Nash Equilibrium in 2-player games and 2-player collaborationist games in his doctoral thesis. VENTSELL (1965) developed solutions for 2x2, 2xn, nx2 and mxn games in game theory. He referred solution with linear programming and found the solution for the games which is a matter of lack of adequate information. OZDEN (1989) resorted to predict the future with the data obtained through the past and with the quantitative decision making techniques. KREPS (1991) made an economic MODELLING with the game theory. MIROWSKI (1992) investigated the history of the economic policy and the emergence of the game theory. McMILLAN (1992) explained the use of game and strategy for senior management by exemplifying them within the game theory. OZDIL (1998) exemplified the place of the game theory in the solution of economic problems in financial market with an implementation. ESEN (2001) analyzed full information static games within the frame of the game theory and applied oligopoly examples. CETIN (2001) made his thesis on the game theory that offers solutions to the problems in the implementation of cooperation which protects the economic and judiciary freedom. CAGLAR (2002) analyzed the history of the game theory and created up-to-date examples. KAFADAR (2002) did his thesis on strategic foreign trade policy and technology transfer. NAEVE (2004) showed that in the game tree, every branch has a knot, a decision and so each knot can have more than one strategy. OZER (2004) applied the game theory in agriculture. ORAN (2004) exemplified the game theory with current events. GREIF (2005) made a historical analysis of the game theory for the economics. RAGHAVAN (2005) created 2-player zero-sum games. SUN and KHAN (2005) analyzed complex strategies for non-zero-sum n>2-player games and fastened them in Nash Equilibrium. CHARTWRINGHT (2009) analyzed and exemplified the balance in multiplayer games for simple strategies.

In this study, a game theory whose first foundation was laid in 1838 and then has attracted more and more interest and have been analyzed a lot was analyzed.

With the developed science and the technology as a result of this science, as the human kind is at the peak of his knowledge level, the human are aware of many more things and the need to consider all of these has made his each step in life even harder.

Now, when deciding about anything, we have to consider the variables apart from us. The game theory provides the individual a binocular evaluation by calculating the variables apart from himself that may affect himself, apart from a one-eyed evaluation by only considering their situations. If the decisions to be made, the behavior to be applied vary according to what others do or will do, the game theory provides solutions to these situations. When looking at the subject about which a decision will be made with a magnifying glass, it would be a practical tool for the individual to make a healthy and more precise decisions. As it makes the analysis of quiet complex situations easier, it is an essential knowledge for both daily life and work life.

In this study, the types of the game theory with its notions and hypotheses were mentioned, two-player zero-sum games and generally well accepted solution methods were focused on. In addition, 3-player non-zero-sum games were defined and exemplified.

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